Mathematics for Europe

The online consultation on mathematics was carried out from 29 January to 15 May 2016 by the European Commission Directorate General for Communications Networks, Content & Technology (DG CONNECT) in the context of a stakeholder consultation to prepare the Horizon 2020 Work Programme 2018-2020.

June 2016

FOREWORD

This report presents the results of the online consultation on mathematics[1] launched early 2016 to nourish the future Horizon2020 Work Programme (2018-20) with innovative mathematical content.

Mathematics is recognized today as essential and indispensable for addressing the major challenges in science, technology and society. Faced with the abundance of data on social, technical, economic, ecological, and technological systems new and sophisticated mathematical tools are required for these data to help us tackle pressing societal challenges and provide us with the necessary technological advantages.

Whereas mathematics is not a prerequisite in Horizon2020, the various areas covered by the programme rely on its development and its use; HPC, Big data, Quantum computing just to name a few. Without mathematical tools, future research will be severely hampered.

The objective of the consultation was to listen to mathematicians and explore the current and emerging challenges in the field of mathematics. What are the emerging key mathematical areas? What is the potential of mathematics for Horizon2020 and beyond?

The response was overwhelming with over 180 contributions covering a large variety of fields, submitted by an impressive set of mathematicians from Europe and the world. The results of the consultation confirm that there is enormous variety of mathematical fields and a huge potential for the European mathematical community.

It also confirms the message already conveyed by the consultation and workshop on Mathematics and Digital Science carried out in 2014: Mathematics is at the base of modern science, it is essential for the development of computers, the crunching of data, and the research requiring computing power and exploiting big amounts of data.

The results of this consultation will feed into the future Horizon2020 Work Programme. The report will be presented at the ICT Proposers Day in September 2016 to enable proposers of future Horizon2020 Work Programme to meet mathematicians and make them aware of the potential of mathematical competence in Europe.

However, one report can do little by itself. The European mathematicians need to be active, to raise awareness of the ground breaking role of mathematics, and to ensure that the new and emerging mathematical fields as well as the new implementation possibilities of existing mathematics are fully exploited for the benefit of the European Science and Innovation. The discussion does not close with this report, it has just started.

I would like to express my appreciation and thanks to Anni Hellman for launching the debate by means of the consultation and for writing this instructive report.

Thank you to everyone who has contributed and for the compilation of the results.

Thierry Van der Pyl

1   https://ec.europa.eu/futurium/en/content/open-consultation-mathematics-horizon-2020-next-work-programme

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1. EXECUTIVE SUMMARY

As some contributors so eloquently express, mathematics can be seen as a soil in which other sciences, technologies and applications are deeply rooted. If we want good fruit and harvest, the soil must be cared for. The future sciences will need new soil to grow, they will need new mathematics.

Current developments in big data analysis and HPC's mathematical basis have shown that existing areas of mathematics formerly deemed merely theoretical, such as algebra and topology, are now very important for these fields. It is therefore impossible to predict what mathematical areas should be in the focus now to be exploitable in 50 years. The mathematicians will be key in identifying the potential of emerging and existing mathematical fields for, amongst other areas, the development of exascale and quantum computing, analytical and simulation tools for big data and for the modelling needed to meet the future environmental, societal and industrial challenges.

Mathematics is a prerequisite for contemporary development in most fields of science. It is essential for both the exploitation of big data and the development of HPC towards exascale and quantum. At the same time, these developments are also enablers for new mathematics.

The consultation received 181 responses covering a large variety of mathematical disciplines. It is to be noted that this consultation did not aim at covering all fields of mathematics, nor do the contributions do so. The consultation was a fully open, public consultation addressed to professionals who could contribute on a voluntary basis .

Many important areas are covered in the contributions received such as data analysis, topological applications, mathematics for HPC, and MSO (Modelling, simulation and optimisation). Biomathematics is also emerging as a new prominent area.

Several contributions also address the challenges of the sizes of European Commission grants and the difficulties in matching them with the mathematical and computer science research needs. The concern is that the present format of European funding will not allow Europe to retain its mathematical talent.

The contributions can be roughly categorised as follows into areas:
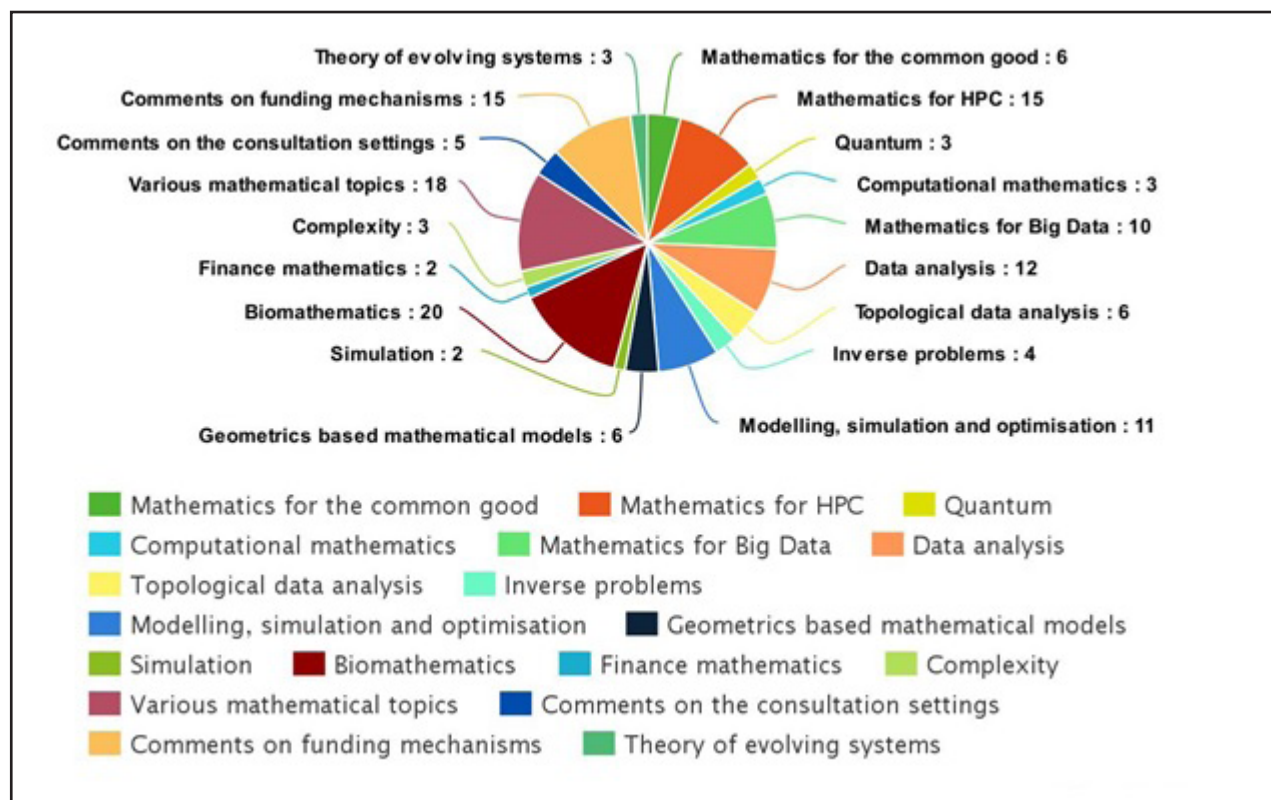
The content of contributions is varied: some concentrated on new and emerging mathematical areas needed for the development of HPC, data or better simulation tools. Among these, we find tropical mathematics (algebra and geometry), algebraic-topological-geometrical methods for data analysis, approximate inference methods, stochastic geometric mechanics, integral biomathics, theory of evolving systems, etc. Others pointed at existing mathematical fields that can be exploited for new solutions, such as topology, optimisation and stochastic processes. Problems requiring mathematics were also suggested: mesoscale modelling, systems biology, CAD/CAE in engineering, etc. Quite importantly, the need for collaboration and increasing convergence between different mathematical disciplines as well as between mathematics and other sciences is emphasised in many contributions.
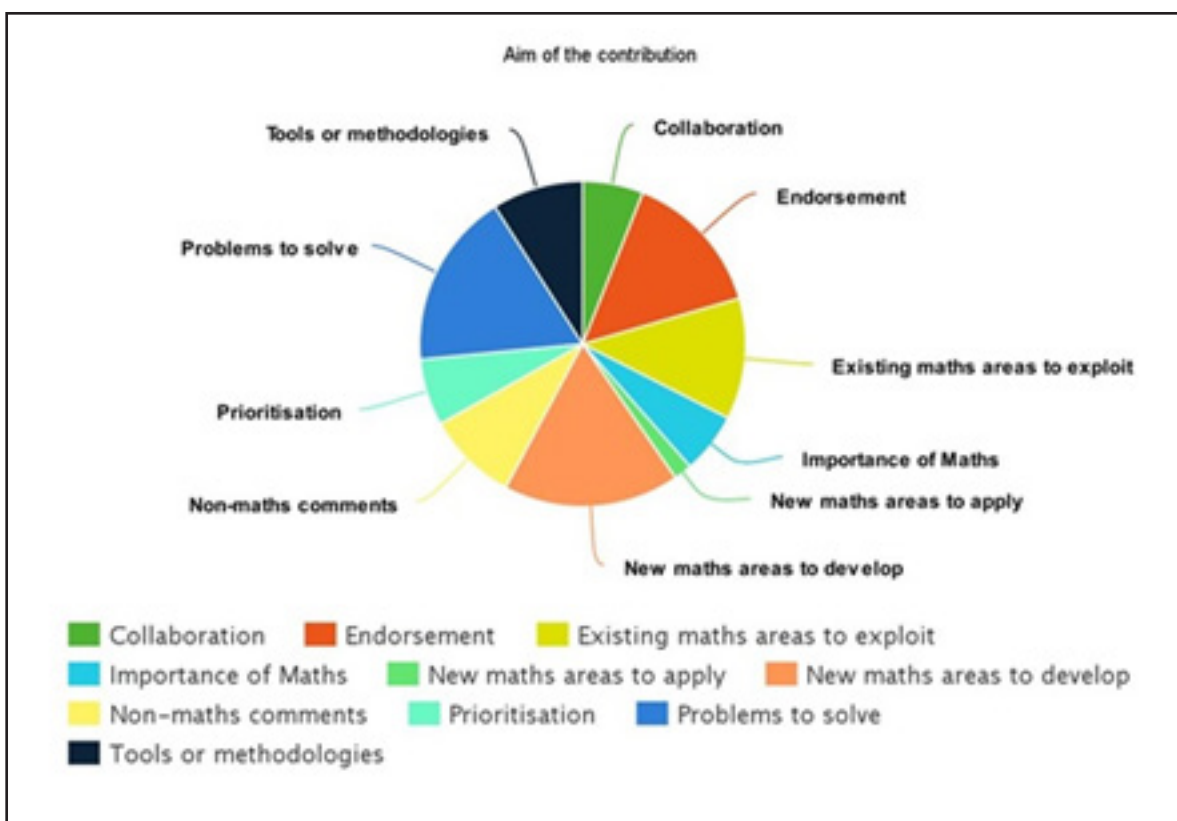
Because the online consultation did not equally reach all mathematical communities, some important areas are missing from the contributions and others are only mentioned. This does not make them less essential or the consultation less noteworthy.

The results demonstrate the importance of mathematics as a whole. Undoubtedly fields such as probability, statistics and financial mathematics, which appear with just a few mentions in the contributions, are indispensable, with new challenges requiring new approaches. Blockchain, trading algorithms, actuarial challenges of tackling big data and its implications for needs for inverse probability theory approaches, are certainly important for the society at large.

Artificial intelligence, robotics and complexity theory as a whole are also fields that require very high level mathematical input. They have been scarcely mentioned in the contributions but this does not mean that they are less important for Europe and for the European Science and Innovation.
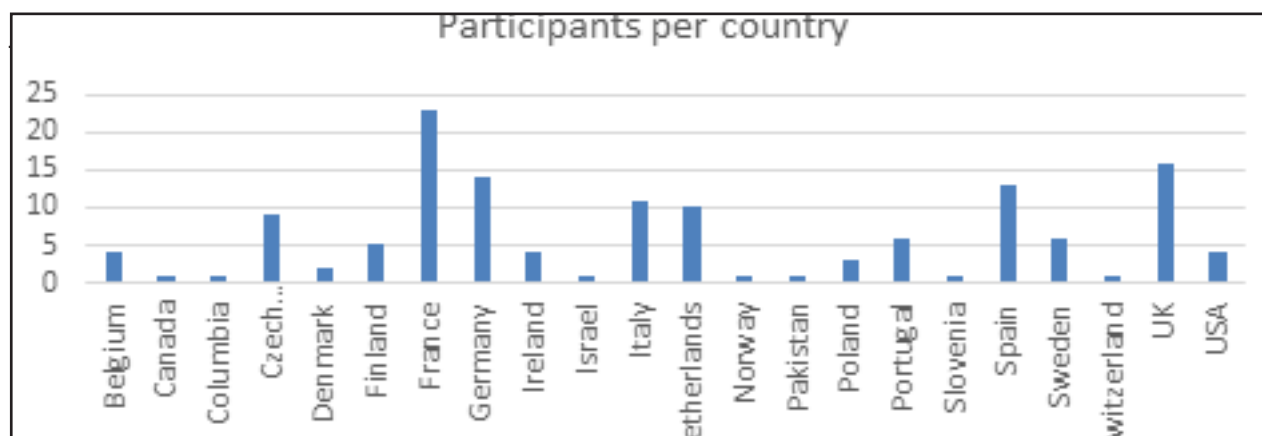
The conclusion is clear: Mathematics is a vital part of the European research and innovation landscape. By playing a bigger role in the Horizon2020 calls as a horizontal, cross cutting competence, mathematics would certainly increase the scalability and the quality of the projects. Contributions received also show that there is a real potential for the FET proactives to exploit relevant mathematical fields.

2.   BACKGROUND

Mathematics is recognized today as essential and indispensable for addressing the major challenges in science, technology and society. New and sophisticated mathematical tools are required to process the abundance of data on social, technical, economic, ecological, and technological systems, and help tackle pressing societal challenges while enabling the necessary technological advances.

The overall objectives of the consultation were to inform the future Horizon2020 Work Programme (2018-20) with innovative mathematical content, and to understand how mathematics could contribute to European research, especially in the area of High Performance Computing, Big data analysis, and in the development of the Digital Single Market[1].

Concrete and upstream questions were asked: what are the new areas emerging in mathematics that are definitely worth investing in at European level? Where can we expect to produce ground breaking outcomes for Europe? What can mathematics do in key areas such as HPC and big data analysis? What are the other vital areas of mathematics to consider?

Additionally, some areas were identified as a starting point to the discussion: topological data analysis and other potential mathematical methods for big data analysis, mathematics for HPC, exascale and exabyte, quantum algorithms, high-dimensional inference problems, secrecy and privacy.

The consultation was preceded by another online consultation and a workshop on mathematics in 2014[2]. Whilst the results of this earlier consultation widely remain valid, new areas emerged such as, among others, lively discussion and high level contributions on biomathematics and various data analysis methodologies. These results strengthen the conclusions that mathematical presence would be clear added value in various fields of science and technology.

The consultation is part of a general stakeholder consultation carried out during the first half of 2016, in view of the process towards the Horizon2020 Work Programme 2018-2020. This report presents an overview of the results of the consultation. The individual contributions can be read on the Futurium platform[3].

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1   https://ec.europa.eu/digital-single-market/en/digital-single-market
2   https://ec.europa.eu/digital-single-market/en/content/mathematics-and-digital-science
3   https://ec.europa.eu/futurium/en/content/online-consultation-mathematics-excellence-science-horizon2020

The contributions received show that Europe can lead in mathematical applications for big data, computing and especially HPC, and that they can support the European ambition to be a leader in modern science and innovation.

3.   MATHEMATICS IN EUROPE TODAY

The latest publication by the European Science Foundation on "Mathematics, its use and benefits for Europe"[4] highlights success stories on the collaboration of mathematics with industry. By means of mathematical algorithms, applied theories and adapted software, remarkable improvements have been possible in the design, planning and implementation phases of the industrial or service processes. This has lead to considerable savings and an increase in productivity. Various areas are listed:

• automotive industry (software for simulation of spray painting, improved automotive injectors, designing oil filters, accelerated simulation, acoustic car design, computation of optical free-form surfaces);

• manufacturing (modelling the instabilities in aluminum reduction cells, numerical simulation for the aluminum industry, imbalance estimation in rotating machinery, modelling and optimal control of chemical mechanical planarization, anti-reflection coatings, modelling plasma PVD, improvements in semiconductor crystal production, polysilicon fuses, numerical simulation of metallurgical processes in silicon production, optimal utilization of colored gemstones, simulation of polymeric textile products, modelling and analysis of rotary fiber spinning);

• aerospace and electronics (modelling and simulation of spacecraft charging, flow control of an air duct, aero-engine nacelle acoustic treatments, reduction of testing by developing new ways of simulation, low cost airborne laser fly, simulation of stochastic radar signals, optimization of satellite coverage, simulating rowing boats, waves for ship-simulation, better prediction and understanding of rogue waves, sails modelling, mathematical modelling of complex materials in underwater sonar systems, optimization algorithms in electronics design automation, mathematical modelling of charge transport in semiconductors, online simulation of 3D nano-optical components);

• energy (bentonite buffer in nuclear waste management, intelligent video understanding applied to plasma facing component monitoring, modelling coal combustion, optimal Flames in industrial furnaces, optimizing a complex hydroelectric cascade in an electricity market, mathematical models for oil pipelining, improving the simulation of multiphase flows in pipelines, near real-time FCC riser simulation and visualization, intelligent agent based modelling and simulation of electrical grids, a kinetic model of blast furnace automation, numerical modelling of heterogeous porous media, aero-acoustic virtual design of exhaust systems);

• environment (numerical simulation for environmental prediction, predicting climate change, solving underground water problems, modelling and simulation of environmental problems, new storm surge forecasting model in the Netherlands, simulation and optimization of waste water filtration, solar reflector design, CELMEC, simulation of a moving bed reactor used in the pulp and paper industry, dynamic image based lighting for highly realistic lighting in building design);

• health (optimization of radiation therapy, simulating atrial fibrillation, realistic modelling of human head tissues exposure to electromagnetic waves, mathematical modelling of a decontamination process, mathematical modelling of an ultrasound sensor for bioprocesses, minimal paths and virtual endoscopy, simulation of a bone- prosthesis system, unravelling the genetic code, forecasting for urgent medical care call centres, three-dimensional X-ray imaging for dentists, computer simulations in electrocardiology, Non Fickian diffusion in polymers and medical applications, statistical testing of molecular motion inside living cells, non-invasive test for monitoring diabetes, Qlucore-bioinformatics, design of innovative nonwovens to be used as insole in functional shoes, early cancer detection by proteomics fingerprinting);

4   http://www.esf.org/fileadmin/Public_documents/Publications/mathematics%26industry.pdf

- services (tool for population pharmacology, shopper behavior modelling, generation of assignments of products to consumers, geotemporal exploitation and analysis platform, analysis and forecasting the evolution of human potential and the job market, the use of queuing theory to increase the effectiveness of physician staffing in the emergency department, air elimination in milk pump systems, optimal portfolio mix using insurance market data, new approaches to reinsurance risk calculation, use of mathematics for transport networking, improving cable network capacity, making smart phones even smarter);

- transport and logistics (optimal trip planning in the presence of random delay, aircraft icing solutions, optimization of the emergency management systems, optimization of public transport systems, forecasting model selection in fast moving consumer goods supply chains, increased efficiency in air traffic control, oscillation-free positioning of a container crane in a high rack warehouse, modelling vibrations and noise in complex built-up structures with a wave chaos approach, truck load analysis, calibration procedure for a height measurement system for excavators, crew rostering (scheduling), preventive maintenance optimization of trains air conditioning systems, optimization in sea logistics, control of navigable rivers, liner network modelling, contributing to the building of the digital society);

- other domains (optimal financial portfolios, financial derivatives pricing, pricing model of catastrophe bonds with complete calibration procedure, quantifying the liquidity premium in bond spreads for insurance liability valuation, modelling and forecasting stock price behaviour in high frequency trading, the resource valuation and optimisation model, realistic assessment of financial products, model based optimum design of experiments, integral equation method, evaluation of dilatometer experiments, solution and model appraisal in reservoir inverse problems using global optimization methods, high-speed large integer arithmetic, modelling and assessment of maintenance efficiency of repairable systems, optimization of electricity production, optimized needle boards based on simulations of needle punch patterns, virtual piano based on mathematical modelling, secure communication for automatic teller machines, uncertainty assessment in high-dimensional nonlinear inverse problems, modelling and detection of realistic human actions in video, real-time video distortion correction, cognitive vision, handwriting recognition).

Interestingly, the largest computations in fluid mechanics in the world and one of the biggest computational capacities nowadays is in Hollywood (US). The film industry is running enormous simulations of flows, for instance to scenify realistic-looking sea, waves, rivers, or to imitate real-life behaviour and produce other remarkable visual effects.

4. COLLABORATION, CONVERGENCE AND INTERACTION

The future of mathematics lies in collaboration with state-of-the-art sciences. New converging fields of science and mathematics are being conceived. Equally important, many areas within mathematics are converging and increasingly interacting. Indeed, substantial interaction amongst mathematicians and with other scientific disciplines is required to meet the needs of data driven science and modern innovation.

Collaboration will allow for existing scientific fields to be reinforced and for new scientific areas to emerge. Widely emphasized in the contributions received, biomathematics appears to be promising. Interaction with chemistry seems to have huge potential as well, especially for quantum chemistry and physical chemistry.

Other areas proposed for intensified interaction with mathematics are:

1. Life sciences, biology, health and medicine have progressed and offer a broad potential for innovation.

2. Computer science in general, and HPC and big data in particular, are already well identified and require combined efforts. Statistical learning and deep neural networks have raised important challenges – in fact, the future of artificial intelligence research lies herein.

3. Physics at all scales and engineering sciences are source of innovation. Material science using numerical applications for modelling with HPC will also lead to new developments.

4. Chemistry should not be underestimated.

5. Social sciences, in the broadest sense, is an important field for investment by mathematics, via the development of complex systems or modelling and data analysis..

In general, the analysis of heterogeneous data such as unstructured data sets or image data is relevant for most fields of science, and requires very sophisticated mathematical tools – and indeed collaboration of several mathematical fields.

In today's mathematics, it would be artificial and counterproductive to draw a boundary between theory and application. It is preferable to ensure a continuum between theoretical and applied mathematics and overcome the siloed approach of traditional mathematics. To this aim, mathematicians need training, incitation and incentives, and the creation of interdisciplinary and inter-mathematical teams to stimulate new ways of collaborating.

Contributions mentioned the cases of HPC and big data where specific mathematical developments apply:

•   The interface between logic, computer science, algebraic topology;

•   Statistics, learning, optimization and approximation in high or infinite dimension;

•   PDEs and control theory, combined with stochastics, big data, geometry, topology;

•   Randomized algorithms in relation with optimization and functional analysis;

•   Numerical analysis and algebra for high performance and parallel computing.

5.   CHALLENGES

There is a huge range of challenges that mathematics could help tackle: from the worldwide question of"how can quantum algorithms solve the world's problems?" to industrial application areas and mathematical problems yet unsolved.

The listing below contains some challenges presented in the contributions.

•   Inverse problems are quite important now, with the availability of data and results often preceding the reasons for them. These challenges relate to health sciences (EEG, tomography), mechanics and industry to test the reliability of metal pieces in planes for instance and actuarial sciences, where risk probabilities may be detected from big data, but the risk profiles are to be identified behind them.

•   There is a need to develop efficient algorithms/approaches for decision making and optimisation of big data sets. The scope for looking at how decision making under risk and uncertainty could be extended to contribute more to crime prevention, climate change, migration and defence.

•   Image data manipulation and analysis is a very contemporary problem: While imaging and reconstruction methods are improved continuously, quantitative analysis of the resulting data lags behind.

•   Data reduction to allow real-time monitoring of data streams (including efficient algorithms for sparse high-dimensional data), efficient ways to combine data sources (data fusion, inclusion of covariate data) and handling different forms of data like images, and monitoring the monitoring system is a very present-day mathematical challenge.

•   Quantum technology, when more developed, will jeopardize today's security and cryptography methods, including blockchain technologies. New mathematical methods, tightly coupled with the underlying physics, are vital here.

- Mathematical methods are needed for validation, verification and quantification of uncertainty, essential in complex and CPU-time expensive measurement processes or computer codes.

- The construction of models and simulation of natural phenomena as well as the improvement of engineering processes is increasingly facilitated by the availability of computing resources and large amounts of data. However, most physical phenomena of interest today operate across large ranges of scale, making conventional computational methods simply useless. Complexity of multi-physics, such as multi-phase, multi-discipline, highly nonlinear and evolving domains and interfaces calls for a new generation of computational methods;

- The mechanism of biological invasions and the study of competitive species require tools from stochastic modelling, scientific computing and other fields of applied mathematics.

- Physics-based modelling is focusing on early phases in the life cycle of industrial products and systems (design, engineering and commissioning) whereas data-based modelling is focusing on later phases (operation and service). Combining physics and data-based approaches to provide new solutions and services is a key challenge for mathematics.

- Biomathematics is a field where various existing mathematical areas will be the basis of new implementations and applications.

- How can we reuse / adapt engineering models in a (hard) real-time context on standard hardware required for operation? How can we realize hybrid algorithms using the best of both worlds? How can we reuse data to improve engineering models? How can we realize product-integrated simulation algorithms (simulation inside) rather than rely on limited HPC resources? How to use a combination of product-integrated computation and HPC in the cloud? Providing efficient answers to these questions has the potential to open up a new era of predictive solutions and services.

Other mentioned areas are: mathematical models for solar power towers, mathematical modelling for environment and sustainable development, that are brought up as important application areas as well as uncertainty quantification (UQ) as a link between applied mathematics, statistics and real world applications. Mathematics of mesoscale systems are important for material sciences as well as atmospheric research and fluid mechanics, to which also the new field of stochastic geometric mechanics presents additional mathematical challenges. Validation, calibration, integration are important areas for industry, the study of dynamical systems partakes of pure mathematics but has numerous applications in real life.

To stimulate the work of mathematricians in these vital areas, a possibility would be to create a "prize" delivered to the first that finds a solution to one of those crucial problems.

The list above is not an exhaustive presentation of the contributions, but it shows the wealth of areas where mathematical development and contribution is clearly required and where mathematics can make a real difference in research or innovation.

6.   MATHEMATICS FOR INDUSTRY AND INNOVATION

The digitalization of industrial processes including planning, production and development is at the core of industrial growth. And its mathematics that provides models, analysis and solutions for these processes.

The computational tools in high performance computing (HPC) are a fundamental part of industrial growth. Modelling of the physical processes of the production allows virtual prototyping and hence shorter and more cost effective developing cycles.

Several disciplines of mathematics play an important role in these tasks, such as (partial) differential equations, optimization, numerical analysis, control, or probability and mathematical statistics. Since recently, the methods of data science are also used in production.

Mathematics and Computer Science go hand in hand in the innovation value chains, for instance in HPC and ICT. Often, mathematics provides algorithms and methods while Computer Science provides the tools to innovate.

7.   MATHEMATICS FOR HPC

Exascale computing is the expected next step in the long road of exponential performance increase that has started over 50 years ago. HPC systems continue to scale up in compute node and processor core count. New mathematics, mathematical methods and tools are required both in the development process, and to make the most of this increase and harness it to tackle new and more complex problems and crunch increasing amounts of data.

New mathematics will be required to implement contemporary multicore processors, computers and high-performance systems. Indeed, sheer computational power will not be enough as the computational power of future exascale-computers will be based on massive parallelism requiring new solutions to be developed. New methods can be developed to deal effectively with massive parallelism, combining ideas from numerical analysis, high-performance computing, probability and inference, complementing the classical deterministic methods such as iterative methods for linear and non-linear systems. This approach would help simplify simulation and optimization methods and lead to a better integration of existing data into simulation runs (data driven simulation).

Algorithms are needed to address the engineering and very large-scale matrix problems arising with exascale computing. Developing and analyzing the methods and algorithms will require substantial mathematical effort.

The landscape of parallel computers is dynamic and quickly changing, and mathematics needs to keep up with this. Different methods turn out to be optimal for different scales of HPC and deserve our continuous attention, research efforts and support. All in all, the mathematical development is crucial to fully harness the capacities of these computing powers.

There is also an increasing need to tackle and solve both data intensive and compute intensive problems. Scalable mathematical methods and corresponding scientific algorithms for multi-petaflop and exa-flop systems must be fault tolerant and fault resilient, since the probability of faults increases with scale. Resilience at the system software and at the algorithmic level is needed as a crosscutting and co-design effort. Additionally, employing novel mathematics and algorithms can lead to a substantial improvement in the performance of important applications. More mathematical effort is necessary.

Essential scientific and industrial applications, such as those applying modelling, simulation and optimisation methods (MSO), require novel mathematics and mathematical software that address the scalability and resilience challenges of current- and future-generation extreme-scale HPC systems. However, there is still a growing gap between the specialized machines and programming tools aiming to reach this goal, and what many simulations can efficiently use. Often there are limitations to scalability inherent to the numerical methods.

As regards the resources required, the collaboration between computer scientists and mathematicians is crucial to achieve the new era of exascale computing and the mathematical development that is a prerequisite for the exploitation of these future computers. Moreover, the role of statisticians will be important, to quantify uncertainty in new high-performance simulations, and thereby assess their accuracy in a reliable way.

HPC technology is also a prerequisite for mathematics of large scale problems. Accordingly, mathematics should not target only the efficient use of the HPC systems but should target primarily the large scale problems themselves.

The earlier online consultation that was carried out in 2014 on Mathematics and Digital Science, asked how mathematicians can help Europe advance towards data-centered HPC. The consultation and the workshop that followed concluded that the European Commission should encourage a focus on the development of mathematically-informed algorithms and designs with core technical issues. Some of these aims have already been taken up by the ETP4HPC, the European HPC Technology Platform[5], in its strategic proposals to the Commission on the European HPC development. However, a lot more remains to be done.

In the strive towards exascale – and mathematics to support it – a lot has already been done but many difficult issues are still open. Moreover, there is still ample room for collaboration between mathematicians and computer scientists.

8.   QUANTUM

Quantum technologies promise new, exciting ways to solve complex problems. Indeed, quantum research is at the crossroads of mathematics, theoretical physics and computer science. However, quantum technology is not sufficiently robust yet to deliver

Quantum machine learning can provide exponential speed-ups over classical computers for a variety of learning tasks by reducing computing times to logarithmic. DARPA, NSF, Google and MIT are currently investing strongly and studying these themes. Quantum algorithms are generally applicable to certain classes of optimization problems (this links to MSO). In addition, quantum information and security are vital for secure communications and a deep theoretical analysis is needed to examine the capacities and limits of protocols.

A real life quantum computer and related solutions will nevertheless also present a concrete risk to many contemporary security protocols, from military applications to blockchain. Cryptographical solutions based on the impossibility to solve them within reasonable time frames may need rethinking or replacing with new quantum-proof algorithms.

9.   DATA ANALYSIS

Large data sets are part of our everyday life. Huge amounts of data are being created constantly in all areas of life, from sensors to social media. Data analysis, and especially methods to crunch big and heterogeneous data and to obtain new information from the abundance of data, are amongst the most important areas of today's mathematical needs.

Mathematical methods for identification of patterns, anomalies and trends in data masses are in a key position for data driven science, and for applications in engineering, life sciences, financial sciences and numerous other fields. These are often inspired by machine learning and artificial intelligence research.

Algebraic, geometric mathematics, data and computational topology, topological data analysis linked to high dimensional inference, combinatorics and statistics have vastly developed in the last few years to tackle the big data challenge. This challenge continues and is growing and methodologies will need to develop further to keep pace with these requirements.

Inverse problems are logically part of the challenges of big data. In fields of life sciences, engineering, actuarial sciences and various other fields we are able to detect anomalies, trends and patterns, but identifying the underlying reasons and dependencies poses challenging mathematical problems.

Major technological advances in sensors and mobile devices have led to the availability of high volume – high frequency data originating from technical and social networks. These data streams not only contain information about directly measurable quantities but also about changes in their structure. Timely detection of structural changes of such networks is of paramount importance since our daily life (both social and

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professional) depends on these networks. Examples include detection of computer network intrusion and terrorist attacks. Big data analysis is one of the areas where mathematics has the clearest and most direct link to everyday life.

Conclusively, big data analysis, and its key fields such as inverse problems and topology are crucial to allow Europe to turn the data it harvests into jobs, growth, profit and scientific results.

10. MODELLING AND SIMULATION METHODOLOGIES

The challenges of our modern society show rising complexity and their solution processes, increasingly requiring a holistic approach. It is necessary to provide decision makers with tools that allow long-term risk analysis, improvements or even optimization and control.

One of the key technologies in this process is the use of mathematical methods, in particular modelling, simulation and optimisation methods (MSO), which have proven effective tools for solving many problems, e.g. energy production and transportation, risk evaluation and pricing, finance modelling, realistic prediction of wind fields, solar radiation, air pollution and forest fires, prediction of climate change, improving the filtration process for drinking water treatment and optimization methods for intensity-modulated radiation therapy.

These methods are highly complex and are typically processed via the most modern tools of informatics including high performance computing (HPC) and access to big data bases and usually need support of skilled experts. These experts are scarce, in particular in small and medium enterprises, or they desert Europe. Mathematical modelling, simulation and optimization (MSO) is a central cross-cutting scientific approach which today forms the third pillar of science besides experiment and theory (although in physics and chemistry sometimes this is part of theory). In recent years mathematicians together with their partners in application fields and computing have made great efforts to improve the mathematical technologies involved in MSO. Examples are the great achievements in areas such as model reduction, adaptive solution of partial differential equations, compressed sensing, or image and signal processing. The field of MSO is broad and includes so wide topics as, for instance, many important fields of biomathematics, biostatistics, computational chemistry, computational engineering, finance, insurance and data science.

Mathematical techniques have revolutionized the scientific development of some areas of science and technology, but as a transversal research field MSO is seldom seen in European funding schemes.

11. BIOMATHEMATICS

Biomathematics, also referred to as "mathematical and theoretical biology", "mathematical biology" or "theoretical biology", is an interdisciplinary research field with wide applications in as biology, biotechnology and medicine. Various mathematical tools and techniques can be deployed to better understand biological phenomena, e.g. in pharmacology, physiology, toxicology, immunology, ecology, evolution, oncology, natural resource management, etc. In the past years, there has been a substantial increase of interest in this field. This could be attributed to advanced analytical tools, computing power expansion and to a recent development of other mathematical tools which facilitate further research.

The participants compared the rising interaction of mathematics and biology with that of mathematics and physics, which holds a long history of research and mutual affection.

So far, mathematical tools have been partially inadequate, since they relied on a single form of mathematics, without considering or applying simultaneity, which is the norm in biology. A paradigm shift in mathematics is necessary to successfully address biological problems and the related mathematical phenomena. Simultaneity or parallelism is said to be achieved through new forms of computing, such as quantum computing.

The proposed areas of future attention include stochasticity, noise effects in biological models, understanding of anticipatory and living systems, morphogenesis and pattern formation, Stefan problems in PDE, percolation models, symmetry problems, genetic algorithms and particularly medicinal research, e.g. in drug discovery and personalized medicine, which could hold great financial and societal implications.

New or adapted analytical tools should be developped to benefit biology and physics. Suggested tools include biostatistics, modelling, quantum computing, Leslie-McKendrick-von Foster demography models, Hodgkin-Huxley action potential model, bioinformatics, data mining models.

12.   OTHER MATHEMATICAL AREAS

Contributions to the consultation mostly focused on a few areas, namely HPC and computing, quantum and big data, MSO and biomathematics but a number of other areas were also mentioned. These include risk, probabilistic, financial and actuarial mathematics, where big data will bring disruptive elements into the approaches. Complexity as a phenomenon related to various areas of life is increasing, and mathematics to model and tackle its challenges needs to develop side by side. This is a question where collaboration between various fields of mathematics with life sciences, social sciences, modern technologies and computer science will be required.

Other interesting fields worth mentioning are: from specific mathematical fields such as tropical mathematics, statistics, fractional dynamics and matrix theory to spectrum of research and decision making within complex societies. Many topics are worthy of collaboration amongst mathematicians in Europe, and some are related to topics of research funded by the Horizon2020 Work Programmes where mathematics certainly could be included. Some could become future areas of active research and interest but as said before, it is impossible to predict which mathematical topics will be at the core of future developments.

13.   SUMMARY

This consultation brought a wealth of information on mathematics and on how it can impact science and innovation in Europe. The potential is obvious and Europe should seize it not only in its process towards Open Science and the Digital Single Market but also in its strive towards innovation, growth and jobs.

Also vividly present in the contributions, is the fact that contemporary mathematics rely on the collaboration between different mathematical fields. Mathematicians must collaborate across mathematical fields as well as across the borders between sciences; they must build bridge with physics, biology, chemistry and increasingly also social sciences and humanities.

Old theoretical fields of mathematics have proven invaluable in the development of high performance computing and when facing the challenges of the data deluge, and new application areas for these well-developed mathematical fields are being identified. Simultaneously new mathematical areas are emerging both from theoretical advancement and to match the newly identified challenges. Mathematics should not only focus on nowadays' applications but should leave room for development, even theoretical, that may well be vital tomorrow.

The wealth of mathematical competence in Europe and its potential for European science and industry is undeniable. The mathematical community should promote its strong potential impact on key societal and scientifical challenges, and the scientific world as well as the industry should involve mathematicians to push innovation further. We cannot know what fields will be essential fifty years from now.

13.   CONTRIBUTIONS RECEIVED

The contributions received and authorised for publication are in a separate document. They appear in their original form, as submitted by the participants. For all contributions, please see the Futurium platform[6].

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6   https://ec.europa.eu/futurium/en/content/open-consultation-mathematics-horizon-2020-next-work-programme

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